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# Dynamics of edge Majorana fermions in $\nu = \frac{5}{2}$ fractional quantum Hall effects

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**Abstract**
Commencing with the composite fermion (CF) description of the $\nu = 5/2$ fractional quantum Hall effect, we study the dynamics of the edge neutral Majorana fermions. We confirm that these neutral modes are chiral and show that a conventional p-wave pairing interaction between CFs does not contribute to the dynamics of the edge neutral fermions. We find an important bilinear coupling between the charged and neutral modes. We show that owing to this coupling, the dispersion of the neutral modes is linear and their velocities are proportional to the wavevector of the charged mode. This dynamic origin of the motion of the edge Majorana fermions has never been predicted before.

## 1. Introduction

Edge states in quantum Hall effects (QHE) play an extremely important role [1]. They are unique known media for reflecting the topological order of the bulk states in fractional QHE (FQHE) [2]. Recently, the topologically protected quantum computation [3] based on the possible non-abelian statistics of quasiparticles in the filling factor $\nu = 5/2$ FQH (EFQH) states [4, 5][1] has attracted great attention, for it provides a possible candidate for the decoherence-free qubit. Several experimental designs for detecting the non-abelian statistics of the $\nu = 5/2$ EFQH state have been proposed [7]. The crucial part of these designs was the point-contact tunnelling of the quasiparticles between different edges of the EFQH droplet.

The fundamental physics behind the non-abelian statistics in the EFQH state is that low energy effective behaviours in the EFQH system are controlled by a $k = 2$ non-abelian Chern–Simons topological quantum field theory in the bulk while the behaviours of the edge states are controlled by a $c = 3k/(k+2) = 3/2$ conformal field theory which consists of chiral Majorana free fermionic modes with a velocity $v_s$ and a chiral free bosonic mode with a velocity $v_n \ll v_s$.

[1] The fractional quantum Hall effect for the even denominator filling factor was observed only at $\nu = 5/2$. For $\nu = 1/2$, there was known to be a Fermi liquid of composite fermions [11]. Experimentally, see Willett *et al* [6] and many further researches. For $\nu \geqslant 9/2$, the states may be the unidirectional density wave states (Fogler *et al* [6]) or anisotropic liquid crystalline states (Fradkin and Kivelson [6], Ciftja and Wexler).

Although this was already recognized by Moore and Read [4] according to the correspondence between the wavefunctions of various FQH states and the correlation functions of the conformal field theory, a fundamental understanding from a microscopic point of view is still lacking. The velocity $v_n$ was not determined. The chirality of the edge modes is a mystery because the composite fermions (CFs) at the half-filling of the Landau level do not see the effective magnetic field.

The present author and his co-workers have provided an effective microscopic theory for the edge states of the odd denominator FQHE [8]. We used a mean field theory with the CF Hamiltonian in the bulk state. Projecting to a given Landau level, the electron band mass is renormalized to the CF effective mass determined by the Coulomb interaction. Shankar and Murthy have provided an understanding of this matter using a Hamiltonian formalism [9]. A simple calculation for the cancellation of the band mass has been given in our previous work, from a Hartree–Fock approximation [10]. However, the random phase approximation calculation showed that the CF effective mass in the half-filled Landau level is logarithmically divergent no matter what gauge is taken [10, 11].

On the other hand, if there is a pairing interaction, this logarithmic divergence of the effective mass of the vortex comes from the extended states with their energy larger than the paring gap [12]. The origin of divergence of the CF effective mass is similar. Therefore, the physics with the energy scale lower than the pairing gap can be studied in the same mean field approximation as for the conventional FQHE if assuming a p-wave EFQH gap for the bulk CFs with the upmost Landau level being half-filled. One can replace the band mass by the Hartree–Fock effective mass. For a pure p-wave superfluid, the pairing interaction gives a finite velocity of the edge Majorana fermionic excitation [13]. However, in this paper, we will show that for the EFQH state, this conventional p-wave paring interaction does not contribute to the velocity of the edge Majorana fermionic modes. We assume that the bulk states have a p-wave gap. By integrating out the bulk states, the effective theory of the edge states includes neutral chiral Majorana fermionic modes and a charged bosonic mode which is described by the Calogero–Sutherland model [14, 15]. Although a conventional p-wave pairing interaction between edge CFs does not contribute to the dynamics of the edge fermionic modes, there is a bilinear coupling between the neutral and charged modes. The velocity of the neutral modes is determined by this important coupling. It vanishes in the ground state as naively expected but linearly increases as the wavevector of the charged mode.

## 2. General description

A two-dimensional interacting spinless electron gas in a high magnetic field is governed by the following Hamiltonian:

$$H = \sum_{\alpha=1}^{N} \frac{1}{2m_{\mathrm{b}}} \left[ \vec{p}_\alpha - \frac{e}{c} \vec{A}(\vec{r}_\alpha) \right]^2 + \sum_{\alpha<\beta} V(\vec{r}_\alpha - \vec{r}_\beta) + \sum_{\alpha} U(\vec{r}_\alpha) + U_{\mathrm{b}}, \qquad (1)$$

where $V(\vec{r})$ is the interaction between electrons. $m_{\mathrm{b}}$ is the band mass of the electron; $U(\vec{r})$ is the external potential trapping the electron gas in a disc and $U_{\mathrm{b}}$ is the interacting potential of the neutralizing positive background charge[2]. The CF theory is a very useful tool in the FQHE physics [11, 17]. We begin with the CF transformation which reads $\Phi(z_1, \ldots, z_N) = \prod_{\alpha<\beta} [\frac{z_\alpha - z_\beta}{|z_\alpha - z_\beta|}]^{\tilde{\phi}} \Psi(z_1, \ldots, z_N)$, where $\Phi$ is the electron wavefunction and $\tilde{\phi}$ is an even integer. We assume that the bulk states have a gap which is caused by a p-wave pairing of CFs for a filling factor $\nu = 1/\tilde{\phi}$ and all gapless excitations are in the edge. We now would like to study the effective theory of the CF edge excitations in a disc. The partition function of the system is

[2] There is an analytic expression for the confining potential $U$ which is important in numerical calculation; see [16].

given by

$$Z = \sum_{N_e} C_N^{N_e} \int_\partial d^2z_1 \ldots d^2z_{N_e} \int_B d^2z_{N_e+1} \ldots d^2z_N \times \left( \sum_\delta |\Psi_\delta|^2 e^{-\beta(E_\delta+E_g)} + \sum_\gamma |\Psi_\gamma|^2 e^{-\beta(E_\gamma+E_g)} \right), \tag{2}$$

where $N_e$ is the CF number in the edge and $N$ is the total electron number. We have divided the sample into the edge $\partial$ and the bulk $B$. $E_g$ is the ground state energy and $E_\delta$ are the low-lying gapless excitation energies with $\delta$ being the excitation mode index. $E_\gamma$ are the gapped excitation energies. One can integrate over the gapped state and the partition function of the system may be written as

$$\begin{aligned} Z &\simeq \sum_{\delta, N_e} C_N^{N_e} \int_{\text{edge}} d^2z_1 \ldots d^2z_{N_e} |\Psi_{e,\delta}|^2 e^{-(\beta E_\delta(N_e)+E_g)} \\ &= \sum_{N_e} C_N^{N_e} \operatorname{Tr}_{(\text{edge})} e^{-\beta H_e}. \end{aligned} \tag{3}$$

In terms of the partition function (3), there is the most probable edge CF number $\bar{N}_e$ which is determined by $\delta Z/\delta N_e = 0$. $\bar{N}_e = \int dx\, \rho_e(x)$ with the edge density $\rho_e(x) = h(x)\bar{n}$. Here $h(x)$ is the edge deformation and $\bar{n}$ is the average density of the bulk electrons. We do not distinguish $\bar{N}_e$ and $N_e$ hereafter if there is no ambiguity. We do not study the bulk physics in the present work and assume that the CF interaction has been renormalized to a weak one. The electron band mass has been renormalized to $m^*$, the effective mass of the CF, which is finite with the order of the Coulomb interaction as we have explained. Hereafter, we use the unit $\hbar = e/c = 2m^* = 1$. For the disc sample with a radius $R$, the edge CFs are restricted to a circular strip near the boundary with its width $\delta R(\vec{r}) \ll R$. The edge Hamiltonian of the CFs reads

$$H_e = \sum_{i=1}^{N_e} [\vec{p}_i - \vec{A}(\vec{r}_i) + \vec{a}_e(\vec{r}_i) + \vec{a}_b(\vec{r}_i)]^2 + \sum_{i<j} V_{\text{eff}}(\vec{r}_i - \vec{r}_j) + \sum_i U_{\text{eff}}(\vec{r}_i), \tag{4}$$

where $V_{\text{eff}}$ is the effective interaction between edge CFs and $U_{\text{eff}}$ is the effective trapping potential including the interaction between the edge and bulk particles. The detailed expression for $U_{\text{eff}}$ may be quantitatively important in numerical calculation (see footnote 1), but here for simplicity, we suppose that the trapping potential is an infinite wall for $r \geqslant R$ in order to analytically study a sharp edge state. Although this may not correctly reflect the quantitative behaviour, the qualitative property which we are studying would not be changed. For real samples, the trapping potential is very much dependent on the sample cleaving. If $U_{\text{eff}}$ is not so sharp, the edge reconstruction is inevitable. In this case, more branches of the edge excitations may appear [18]. The statistics gauge field $\vec{a}_e$ is given by

$$\vec{a}_e(\vec{r}_i) + \vec{a}_b(\vec{r}_i) = \frac{\tilde{\phi}}{2\pi} \sum_{j\neq i} \frac{\hat{z} \times (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^2} + \frac{\tilde{\phi}}{2\pi} \sum_{\alpha \in \text{bulk}} \frac{\hat{z} \times (\vec{r}_i - \vec{r}_\alpha)}{|\vec{r}_i - \vec{r}_\alpha|^2}. \tag{5}$$

Taking the polar coordinate $z_i = r_i\, e^{i\varphi_i}$, the vector potential $A_\varphi(\vec{r}_i) = \frac{B}{2} r_i$ and $A_r(\vec{r}_i) = 0$ and substituting the polar variables and the vector potential into $H_e$, one has

$$\begin{aligned} H_e = \sum_i \Bigg[ &-\frac{\partial^2}{\partial r_i^2} + \left( -\frac{i}{r_i}\frac{\partial}{\partial \varphi_i} + \frac{\tilde{\phi}(N_e - 1)}{2r_i} \right)^2 + \frac{\tilde{\phi}^2}{4R^2} \sum_i \left( \sum_{j\neq i} \cot\frac{\varphi_{ij}}{2} \right)^2 \\ &- i\frac{\tilde{\phi}}{R} \sum_{i<j} \cot\frac{\varphi_{ij}}{2} \left[ \frac{\partial}{\partial r_i} - \frac{\partial}{\partial r_j} \right] - \frac{1}{R}\frac{\partial}{\partial r_i} \Bigg] + V_{\text{eff}} + U_{\text{eff}} + O(\delta R/R), \end{aligned} \tag{6}$$

where we use the mean field approximation $\bar{a}_{b,r} = 0$ and $\bar{a}_{b,r} = -\frac{B}{2} r_i$.

## 3. Solutions

The Schrödinger equation $H_e\Psi = E\Psi$ holds for a FQH edge state with a general filling factor. For a given filling factor, the solution of Schrödinger equation is dependent on the bulk states as we have seen in solving the equations for the odd denominator FQHE [8]. For an even denominator quantum Hall state, e.g. the $\nu = 5/2$ state [19, 20], numerical calculations evidenced that the Moore–Read Pfaffian state may be the ground state [21]. Motivated by the bulk ground state, we can write the general form of the wavefunction as

$$\Psi(z_1, \ldots, z_{N_e}) = \exp\left\{-\mathrm{i}\sum_{i<j}\frac{r_i - r_j}{4R}\cot\frac{\varphi_{ij}}{2}\right\} \times \mathcal{A}(z_1, \ldots, z_{N_e}) f(r_1, \ldots, r_{N_e})\phi_{cs}(\varphi_1, \ldots, \varphi_{N_e}). \tag{7}$$

We anticipate that $f$ determines the chirality of the edge charged mode which is described by $\phi_{cs}$. Then, $\mathcal{A}$ corresponds to the neutral fermionic modes. In fact, Milovanovic and Read have written down the wavefunctions of the edge excitations [13, 22]. We will show that our solutions are consistent with their wavefunctions while we confirm that the neutral fermion modes do indeed have a linear dispersion and are chiral. The new observation is that the velocity of these fermionic modes is proportional to the wavevector of the charged bosonic edge mode.

In terms of the general wavefunction, the problem ready to solve yields

$$\sum_i\left\{-\frac{\partial^2}{\partial r_i^2} - \frac{1}{R}\frac{\partial}{\partial r_i} - \frac{2}{R^2}\frac{\partial \ln\chi_{cs}}{\partial\varphi_i}\frac{\partial \ln\mathcal{A}}{\partial\varphi_i} - 2\frac{\partial \ln\mathcal{A}}{\partial r_i}\frac{\partial}{\partial r_i} + U_{\mathrm{eff}}\right\} f = (E - E_\varphi) f \tag{8}$$

with

$$-\sum_i\frac{1}{R^2}\frac{\partial^2}{\partial\varphi_i^2}\chi_{cs} + \frac{1}{4R^2}\sum_{i<j}\frac{\tilde{\phi}(\tilde{\phi}-1)}{\sin^2(\varphi_{ij}/2)}\chi_{cs} + V_{\mathrm{eff}}\chi_{cs} = E_\varphi\chi_{cs}, \tag{9}$$

$$(\nabla^2\mathcal{A})\phi_{cs} = 0, \qquad \sum_i -\mathrm{i}\frac{\partial\mathcal{A}}{\partial\varphi_i} = \sum_i l_i\mathcal{A}, \tag{10}$$

where $\chi_{cs} = \mathrm{e}^{(\mathrm{i}/2)\tilde{\phi}(N_e-1)\sum_i\varphi_i}\phi_{cs}$ and $l_i$ is a set of integers or half-integers to be determined. Equation (9) describes the charged edge excitations. If one neglects $V_{\mathrm{eff}}$ in equation (9), it is the famous Calogero–Sutherland model with $E_\varphi = \sum_i \frac{n_i^2}{R^2}$, whose solutions are $\chi_{cs} = \Phi(\mathbf{n})\prod_{i<j}[\sin\frac{\varphi_{ij}}{2}]^{\tilde{\phi}}$ where $\Phi(\mathbf{n})$ is the Jack polynomial [23] with the highest weight state satisfying

$$n_i = I_i + \tfrac{1}{2}\sum_{j\neq i}(\tilde{\phi}-1)\,\mathrm{sgn}(n_i - n_j), \tag{11}$$

where the $I_i$ are a set of integers with respect to physical momentum along the azimuthal direction. With the interaction $V_{\mathrm{eff}}$, in the limit of dilute gas, $n_i = I_i + \frac{1}{2}\sum_{j\neq i}\theta(n_j - n_i)$ with $\theta(\Delta n) = (\tilde{\phi} - 1)\,\mathrm{sgn}(\Delta n) - 2\delta_{\tilde{\phi}-1}(\Delta n)$ [8]. The phase shift $2\delta_{\tilde{\phi}-1}(\Delta n)$ comes from the interaction $V_{\mathrm{eff}}$ which is continuous as a function of $\Delta k = \Delta n/R$ and vanishes at $\Delta k = 0$ if $V_{\mathrm{eff}}$ is short range. Because of the step function $\mathrm{sgn}(\Delta n)$, the short range interaction may not affect the low-lying behaviour of the edge modes. However, the velocity of the charged mode is lifted by the interaction [8, 24]. (For details, see [24].) The Coulomb interaction may cause an additional charged branch of the edge excitations with a dispersion $\Delta k \ln\Delta k$ [8], which is precisely the one-dimensional plasmon excitation caused

by the Coulomb interaction [25]. Now, we phenomenologically assume that $V_{\rm eff}$ includes a conventional p-wave pairing interaction

$$H_{\rm pair} = {\rm i}\sum_i \Delta \begin{pmatrix} 0 & \bar{\partial}_i \\ \partial_i & 0 \end{pmatrix}, \tag{12}$$

where a $2\times 2$ matrix acts on the two components of the complex spinless CF state [13]. For a pure $p_x + {\rm i}p_y$ superconductor without the bosonic mode, this p-wave pairing interaction gives a gapless chiral fermion excitation with its velocity $v_n = \Delta$ at the edge [13]. However, for the EFQH state, the pairing interaction may only change the velocity of the charged mode and may not contribute to that of the neutral modes since $\sum_i \partial\mathcal{A}/\partial z_i = \sum_i \partial\mathcal{A}/\partial\bar{z}_i = 0$ according to the total antisymmetry of $\mathcal{A}$ with the exchange of the CFs.

Equation (10) implies that if we can find the eigenstates of the second equation which satisfy the first one, these states will indeed describe the neutral Majorana fermion modes with a linear dispersion. The Moore–Read Pfaffian state ${\rm Pf}(\frac{1}{z_i - z_j})$ satisfies these equations with $\sum_i l_i = -2$. This is the ground state for even edge CF number. The ground state with an odd edge CF is given by $\psi_l = \mathcal{A}(z_1^0 \frac{1}{z_2 - z_3} \ldots \frac{1}{z_{N_{\rm e}-1} - z_{N_{\rm e}}})$. The multi-Majorana fermion excited states are given by [13, 22]

$$\mathcal{A} = \mathcal{A}\left(z_1^{l_1} \ldots z_s^{l_s} \frac{1}{z_{s+1} - z_{s+2}} \ldots\right) = \mathcal{A}\left(\sum \epsilon_{i_1 i_2 \ldots i_s} z_{i_1}^{l_1} z_{i_2}^{l_2} \ldots z_{i_s}^{l_s} \psi_{i_1 i_2 \ldots i_s}^{i_{s+1}\ldots i_N}\right), \tag{13}$$

where $\psi_{i_1 i_2 \ldots i_s}^{i_{s+1}\ldots i_N}$ is a product of $\frac{1}{z_i - z_j}$ where $z_i$ and $z_j$ do not include $z_{i_1}, z_{i_2}, \ldots, z_{i_s}$ and of course it is symmetric for any permutation of $i_1, i_2, \ldots, i_s$.

## 4. The dynamic origin of the neutral fermion velocity and chirality of the charged mode

We now study the dynamics of the neutral fermion and chirality of the edge excitations. As we have seen, the requirement of $(\nabla^2\mathcal{A})\phi_{\rm cs} = 0$ gives rise to the chirality of the neutral Majorana fermion edge modes. However, what if the origin of the motion of the neutral modes for $V_{\rm eff}$ does not contribute to $v_n$? On the other hand, the charged mode at half-filling does not feel an effective magnetic field. The Calogero–Sutherland model has both left- and right-moving gapless modes from the Fermi points. Why is the edge charged mode still chiral? To answer these questions, we consider the radial equation (8). The pseudo-momentum of the Calogero–Sutherland model is defined by $k_i = n_i/R$. The pseudo-Fermi points are in $\pm k_{\rm F} = \pm\tilde{\phi}\bar{N}_{\rm e}/2R$ and low energy excitations are around $k \sim \pm(k_{\rm F} + q)$ for $q \ll k_{\rm F}$. We define the neutral fermion momentum $p_i = l_i/R$ and consider multi-Majorana fermion modes given by equation (13). Notice that $-{\rm i}\frac{\partial\chi_{\rm cs}}{\partial\varphi_i} = \sum_P k_{P_i}\chi_P$ and $-{\rm i}\frac{\partial\mathcal{A}}{\partial\varphi_i} = \mathcal{A}(\sum_{i_1 i_2 \ldots i_s} \epsilon_{i_1 i_2 \ldots i_s}(\delta_{i_1 i} l_1 + \ldots + \delta_{i_s i} l_s) z_{i_1}^{l_1} z_{i_2}^{l_2} \ldots z_{i_s}^{l_s} \psi_{i_1 i_2 \ldots i_s}^{i_{s+1}\ldots i_N})$, where $P$ is a permutation of $\{1, \ldots N_{\rm e}\}$. The fourth term in equation (8) reads

$$-\frac{2}{R^2}\sum_i \frac{\partial \ln\chi_{\rm cs}}{\partial\phi_i}\frac{\partial\ln\mathcal{A}}{\partial\phi_i} = 2\sum_{a=1}^{s} p_a \sum_i k_i C_{k_i a}, \tag{14}$$

where

$$C_{k_i a} = \frac{1}{\chi_{\rm cs}\mathcal{A}}\mathcal{A}\left(\sum_{i_1,\ldots,i_a,\ldots,i_s} \epsilon_{i_1\ldots i_a\ldots i_s} z_{i_1}^{l_1} \ldots z_{i_a}^{l_a} \ldots z_{i_s}^{l_s} \psi_{i_1 i_2 \ldots i_s}^{i_{s+1}\ldots i_N} \sum_{\{P|P_{i_a}=i\}} \chi_P\right). \tag{15}$$

The symmetry of the reflection gives that $C_{ka} = C_{-ka}$. Thus, we have $\sum_i k_i^{(0)} C_{k_i^{(0)} a} = 0$ for the ground state $\{k_i^{(0)}\}$. The low-lying excitations are given by $\{k_i^{(\pm)}\} = \{\pm(k_{\rm F} + q), k_i^0 \neq \pm k_{\rm F}\}$

for $q > 0$. Thus the radial equation reads

$$\sum_i \left\{ -\frac{\partial^2}{\partial r_i^2} + 2k_{\mathrm{F}} q \pm \sum_a 2q C_{\pm k_{\mathrm{F}} a} p_a - 2\left(\frac{\partial \ln \mathcal{A}}{\partial r_i}\right)\frac{\partial}{\partial r_i} + \frac{1}{R}\frac{\partial}{\partial r_i} + U_{\mathrm{eff}} \right\} f = E^{\pm} f. \tag{16}$$

The third term in equation (16) is a bilinear coupling between the wavevectors of the neutral and charged modes. Since $C_{-k_{\mathrm{F}} a} = C_{k_{\mathrm{F}} a}$ which is assumed to be positive, this coupling means that the charged mode around $k_{\mathrm{F}}$ is accompanied by a set of neutral modes while the excitation around $-k_{\mathrm{F}}$ is not physical because the accompanying neutral fermion modes lowered the energy of the system so that it became not lower bounded. The physical excitations are confined around $k_{\mathrm{F}}$. This proves the chirality of the charged edge bosonic mode with its velocity $v_s = v_{\mathrm{F}} = 2k_{\mathrm{F}}$. The neutral modes have velocities $v_{n,a} = 2C_{k_{\mathrm{F}} a} q$. This is an important observation made in this work: $v_{n,a} \ll v_s$ is linearly dependent on the wavevector of the charged mode. This dynamic origin of the velocity of the edge Majorana fermion has never been predicted, in the existing literature.

We emphasize that the origin of the chirality of the charged edge mode for the half-filling factor is very different from that for the FQHE with the odd denominator filling factor. For the conventional FQHE, the edge CFs 'see' an effective magnetic field and the CF cyclotron motion in this effective magnetic field is the origin of the chirality. The CFs for a half-filling factor do not 'see' such an effective magnetic field. The chirality of the charged bosonic edge mode stems from the coupling of this mode with the neutral Majorana chiral fermion modes. Of course, from the point of view of electron motion, the chirality of the edge excitations is still caused by the magnetic field. This is reflected in the wavefunctions: if we reverse the magnetic field, $z$ should be replaced by $\bar{z}$ in all wavefunctions, which leads to a reverse of the chirality of the edge excitations.

## 5. Conclusions

We have studied the dynamics of the edge Majorana fermions for the EFQH state with $\nu = 5/2$. We found that there is a bilinear coupling between neutral and charged modes in the effective edge theory, which determines the dynamics of the neutral modes and the chirality of the charged modes. The velocity of the neutral fermion is proportional to the wavevector of the charged mode. The chirality of the charged edge mode originates from its coupling with the neutral modes.

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## References

[1] Halperin B I 1982 *Phys. Rev.* B **25** 2185
[2] Wen X G 1990 *Phys. Rev. Lett.* **64** 2206
Wen X G 1990 *Phys. Rev.* B **41** 12838
Wen X G and Niu Q 1990 *Phys. Rev.* B **41** 9377
[3] Kitaev A Y 2003 *Ann. Phys.* **303** 2
Freedman M H, Larsen M J and Wang Z 2002 *Commun. Math. Phys.* **227** 605
[4] Moore G and Read N 1991 *Nucl. Phys.* B **360** 362
[5] Greiter M, Wen X G and Wilczek F 1992 *Nucl. Phys.* B **374** 567
[6] Willett R L, Paalanen M A, Ruel R R, West K W, Pfeiffer L N and Bishop D J 1990 *Phys. Rev. Lett.* **65** 112

Fogler M M, Koulakov A A and Shklovskii B I 1996 *Phys. Rev.* B **54** 1853
Fradkin E and Kivelson S A 1999 *Phys. Rev.* B **59** 8065
Ciftja O and Wexler C 2002 *Phys. Rev.* B **65** 045306
[7] Das Sarma S, Freedman M and Nayak C 2005 *Phys. Rev. Lett.* **94** 166802
Stern A and Halperin B I 2006 *Phys. Rev. Lett.* **96** 016802
Bonderson P, Kitaev A and Shtengel K 2006 *Phys. Rev. Lett.* **96** 016803
Hou C-Y and Chamon C 2006 *Preprint* cond-mat/0603142
[8] Yu Y and Zhu Z Y 1998 *Commun. Theor. Phys.* **29** 351
Yu Y, Zheng W J and Zhu Z Y 1997 *Phys. Rev.* B **56** 13279
Zheng W J and Yu Y 1997 *Phys. Rev. Lett.* **79** 3242
Yu Y 2000 *Phys. Rev.* B **61** 4465
[9] For a review, see Murthy G and Shankar R 2003 *Rev. Mod. Phys.* **75** 1101
[10] Yu Y, Su Z B and Dai X 1998 *Phys. Rev.* B **57** 9897
[11] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312
[12] Han J H, Kim J S, Kim M J and Ao P 2005 *Phys. Rev.* B **71** 125108
Han J H, Ao P and Zhu X M 1999 *Preprint* cond-mat/9903125
[13] Fendley P, Fisher M P A and Nayak C 2007 *Phys. Rev.* B **75** 045317
[14] Calogero F 1969 *J. Math. Phys.* **10** 2191
Calogero F 1969 *J. Math. Phys.* **10** 2197
[15] Sutherland B 1971 *J. Math. Phys.* **12** 246
Sutherland B 1971 *Phys. Rev.* A **4** 2019
Sutherland B 1972 *Phys. Rev.* A **5** 1372
[16] Ciftja O and Wexler C 2003 *Phys. Rev.* B **67** 075304
[17] Jain J K 1989 *Phys. Rev. Lett.* **63** 199
[18] Wan X, Rezayi E H and Yang K 2003 *Phys. Rev.* B **68** 125307
[19] Willett R L *et al* 1987 *Phys. Rev. Lett.* **59** 1776
Eisenstein J P *et al* 2002 *Phys. Rev. Lett.* **88** 076801
[20] Xia J S *et al* 2004 *Phys. Rev. Lett.* **93** 176809
[21] Morf R H 1998 *Phys. Rev. Lett.* **80** 1505
[22] Milovanovic M and Read N 1996 *Phys. Rev.* B **53** 13559
[23] Jack H 1969–1970 *Proc. R. Soc. Edinburgh* A **69** 1
Ha Z N C 1995 *Nucl. Phys.* B **435** 604
[24] Wu Y S and Yu Y 1995 *Phys. Rev. Lett.* **75** 890
Wu Y S, Yu Y and Yang H X 2001 *Nucl. Phys.* B **604** 551
Yu Y, Yang H X and Wu Y S 1999 *E-preprint* cond-mat/9911141
[25] Wen X G 1991 *Phys. Rev.* B **44** 5708